UK Junior Mathematical Olympiad 2011

Organised by The United Kingdom Mathematics Trust

Tuesday 14th June 2011

RULES AND GUIDELINES :
READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work. Write in blue or black pen or pencil.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

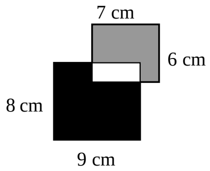
   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

A1 What is the value of $3^3 + 3 \times 3 - 3$?

A2 Two rectangles measuring 6 cm × 7 cm and 8 cm × 9 cm overlap as shown. The region shaded grey has an area of 32 cm^2.



What is the area of the black region?

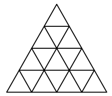
A3 Paul is 32 years old. In 10 years' time, Paul's age will be the sum of the ages of his three sons.

What do the ages of each of Paul's three sons add up to at present?

A4 What is the value of $\dfrac{1}{2-3} - \dfrac{4}{5-6} - \dfrac{7}{8-9}$?

A5 The base of a pyramid has n edges. In terms of n, what is the difference between the number of edges of the pyramid and the number of its faces?

A6 The diagram shows a grid of 16 identical equilateral triangles.

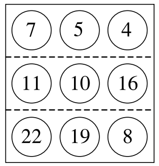


How many different rhombuses are there made up of two adjacent small triangles?

A7 Some rectangular sheets of paper, all the same size, are placed in a pile. The pile is then folded in half to form a booklet. The pages are then numbered in order 1, 2, 3, 4 ... from the first page to the last page.
On one of the sheets, the sum of the numbers on the four pages is 58.

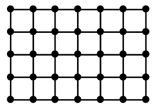
How many sheets of paper were there at the start?

A8 A puzzle starts with nine numbers placed in a grid, as shown.
On each move you are allowed to swap any two numbers.
The aim is to arrange for the total of the numbers in each row to be a multiple of 3.



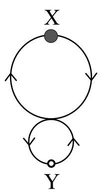
What is the smallest number of moves needed?

A9 The diagram represents a rectangular fishing net, made from ropes knotted together at the points shown. The net is cut several times; each cut severs precisely one section of rope between two adjacent knots.



What is the largest number of such cuts that can be made without splitting the net into two separate pieces?

A10 A 'figure of eight' track is constructed from two circles: a large circle of radius 2 units and a small circle of radius 1 unit. Two cars X, Y start out from the positions shown: X at the top of the large circular part and Y at the bottom of the small circular part. Each car travels round the complete circuit in the directions shown by the arrows.



If Y travels twice as fast as X, how far must X travel before the cars collide?

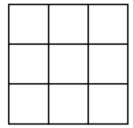
Section B

Your solutions to Section B will have a major effect on your JMO results. Concentrate initially on one or two questions and then **write out full solutions** (not just brief 'answers') using algebra where appropriate.

B1 Every digit of a given positive integer is either a 3 or a 4 with each occurring at least once. The integer is divisible by both 3 and 4.

What is the smallest such integer?

B2 A 3 × 3 grid contains nine numbers, not necessarily integers, one in each cell. Each number is doubled to obtain the number on its immediate right and trebled to obtain the number immediately below it.

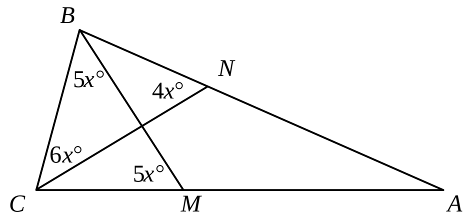


If the sum of the nine numbers is 13, what is the value of the number in the central cell?

B3 When Dad gave out the pocket money, Amy received twice as much as her first brother, three times as much as the second, four times as much as the third and five times as much as the last brother. Peter complained that he had received 30p less than Tom.

Use this information to find all the possible amounts of money that Amy could have received.

B4 In a triangle ABC, M lies on AC and N lies on AB so that $\angle BNC = 4x°$, $\angle BCN = 6x°$ and $\angle BMC = \angle CBM = 5x°$.

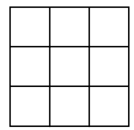


Prove that triangle ABC is isosceles.

B5 Calum and his friend cycle from A to C, passing through B.
During the trip he asks his friend how far they have cycled.
His friend replies "one third as far as it is from here to B".
Ten miles later Calum asks him how far they have to cycle to reach C.
His friend replies again "one third as far as it is from here to B".

How far from A will Calum have cycled when he reaches C?

B6 Pat has a number of counters to place into the cells of a 3 × 3 grid like the one shown. She may place any number of counters in each cell or leave some of the cells empty. She then finds the number of counters in each row and each column. Pat is trying to place counters in such a way that these six totals are all different.



What is the smallest total number of counters that Pat can use?